

# “Point-free” Put-based Bidirectional Programming

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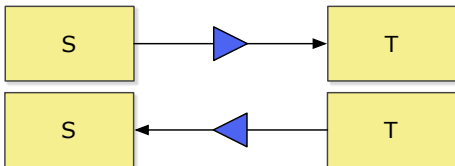
National Institute of Informatics, Tokyo, Japan

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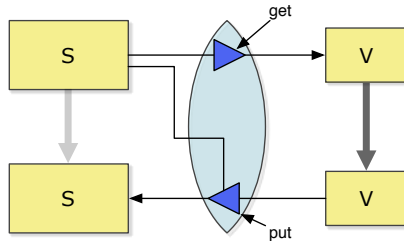
Braga - April 3rd, 2013

# Bidirectional Transformations (BXs)

*“A mechanism for maintaining the consistency of two (or more) related sources of information.”*



- lenses are one of the most popular BX frameworks



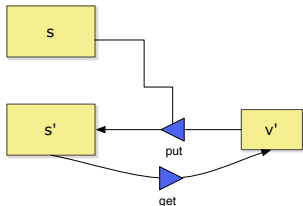
## Framework

```

data  $S \Rightarrow V = \text{Lens } \{ \text{get} : S \rightarrow V$ 
    ,  $\text{put} : S \rightarrow V \rightarrow S \}$ 
    
```

- PUTGET law

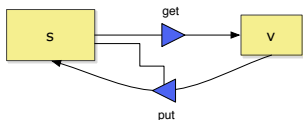
*put must translate  
view updates exactly.  
get defined for  
updated sources.*



$$s' = \text{put } s \ v' \Rightarrow v' = \text{get } s'$$

- GETPUT law

*put must preserve  
empty view updates.  
put defined for  
empty view updates.*



$$v = \text{get } s \Rightarrow s = \text{put } s \ v$$

- BX applications vary on the bidirectionalization approach
- common trait: write *get* and derive *put* automatically
- easy and maintainable
- *get*-based domain-specific lens languages:
  - *put* total (– expressiveness)



J. N. Foster, M. B. Greenwald, J. T. Moore, B. C. Pierce, and A. Schmitt

Combinators for bidirectional tree transformations: A linguistic approach to the view-update problem  
*ACM Transactions on Programming Languages and Systems, 2007.*



H. Pacheco and A. Cunha

Generic Point-free Lenses  
*Mathematics of Program Construction, 2010.*

- *put* partial (– updatability)



D. Liu, Z. Hu, and M. Takeichi

Bidirectional interpretation of XQuery  
*Partial Evaluation and Program Manipulation, 2007.*

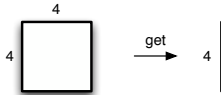


Z. Hu, S.-C. Mu, and M. Takeichi

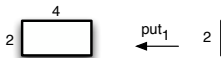
A programmable editor for developing structured documents based on bidirectional transformations  
*Higher Order and Symbolic Computation, 2008.*

# Motivation - Ambiguous *put*

- unavoidable ambiguity: it is well-known that there are many possible well-behaved *puts* for a *get*



$height : (Int, Int) \rightarrow Int$   
 $height(w, h) = h$



-- keep original width  
 $putheight_1 : (Int, Int) \rightarrow Int \rightarrow Int$   
 $putheight_1(w, h) h' =$   
**let  $w' = w$  in  $(w', h')$**



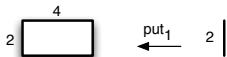
-- keep the width/height ratio  
 $putheight_2 : (Int, Int) \rightarrow Int \rightarrow Int$   
 $putheight_2(w, h) h' =$   
**let  $w' = h' * (w / h)$  in  $(w', h')$**



-- default width  
 $putheight_3 : (Int, Int) \rightarrow Int \rightarrow Int$   
 $putheight_3(w, h) h' =$   
**let  $w' = \text{if } h' \equiv h \text{ then } w \text{ else } 3$  in  $(w', h')$**

## Motivation - An unpractical assumption

- *get*-based programming has an implicit assumption that *it is sufficient to derive a suitable put that can be combined with get to form a well-behaved lens.*
- but **the** most suitable *put* does not exist!
- for *get = height...*
  - shall *put<sub>height</sub>* preserve the width? (rectangle)



- shall *put<sub>height</sub>* update the width? (square)



- each BX approach will provide its own solution!  $\Rightarrow$  boom of BX approaches over the last 10 years

## Lemma

*Given a put function, there exists at most one get function such that GETPUT and PUTGET hold.*

## Theorem (Uniqueness of *get* for well-behaved (partial) *put*)

*Assume a put function such that:*

- 1 *(flip put) v is idempotent, i.e.,  $put (put s v) v = put s v$*
- 2 *put s is injective*

*Then (a) there is exactly one get function such that the resulting lens is well-behaved and (b)  $get s = v \Leftrightarrow s = put s v$*



S. Fischer, Z. Hu and H. Pacheco

"Putback" is the Essence of Bidirectional Programming

GRACE-TR 2012-08, GRACE Center, National Institute of Informatics, December 2012.



# Put-based bidirectional programming

- *get*-based = maintainability at the cost of expressiveness
- write *get* from  $S$  to  $V$

$$S \xrightarrow{f} U \xrightarrow{g} V$$

- however, writing  $put : S \rightarrow V \rightarrow S$  is much more difficult than writing  $get : S \rightarrow V$
- **idea**: language of injective *put* combinators from  $V$  to  $S$

$$S \xleftarrow{f} U \xleftarrow{g} V$$

- *put*-based = describe the behavior of a BX completely!

## Framework

**data**  $S \Leftarrow V = Putlens \{ put : S \rightarrow V \rightarrow S$   
 $, get : S \rightarrow V \}$

# A point-free put-based bidirectional language

- functional languages: **data domain** of algebraic data types
- algebraic data types = trees = sums of products

**data**  $[A] = [] \mid A : [A]$

**data**  $Maybe A = Nothing \mid Just A$

$[A]$   
 $out \downarrow \uparrow in$   
 $Either () (A, [A])$

$Maybe A$   
 $out \downarrow \uparrow in$   
 $Either () A$

- we will build a point-free *put* language that reverses...



H. Pacheco and A. Cunha

Generic Point-free Lenses

*Mathematics of Program Construction, 2010.*

... and is inspired in the injective language from...



S.-C. Mu, Z. Hu, and M. Takeichi

An injective language for reversible computation

*Mathematics of Program Construction, 2004.*

- partial *put* combinators = no updatability problem

## Identity and Composition

$$\text{id} : V \Leftarrow V$$

$$\text{put } s \ v' = v'$$

$$\forall f : S \Leftarrow U, g : U \Leftarrow V. (f \circ g) : S \Leftarrow V$$

$$(f \circ g) \ s \ v' = (\text{put}_f \ s \circ \text{put}_g \ (\text{get}_f \ s)) \ v'$$

## Filtering and bottom

$$\forall p : V \rightarrow \text{Bool}. (\Phi \ p : V \Leftarrow V) \quad \text{bot} :: S \Leftarrow V$$

$$(\Phi \ p) \ s \ v' \mid p \ v' = v' \quad \text{bot } s \ v' = \perp$$

- partial *put*: only certain views are permitted

## Add first element to the source

$$\forall f : (S_1, V) \rightarrow V \rightarrow S_1. \text{ addfst } f : (S_1, V) \Leftarrow V$$
$$\text{put } (s_1, v) \ v' = (s_1', v')$$
$$\text{where } s_1' = \text{if } v' \equiv v \text{ then } s_1 \text{ else } f (s_1, v) \ v'$$

## Keep first element in the source

$$\text{keepfst} : (S_1, V) \Leftarrow V$$
$$\text{keepfst} = \text{addfst } (\lambda(s_1, v) \ v' \rightarrow s_1)$$

- similar for *addrPut*, *keepsnd*

## Drop first element in the view

$$\forall f : V \rightarrow V_1. \text{remfst} : V \Leftarrow (V_1, V)$$
$$\text{put } v (v_1', v') \mid f v' \equiv v_1' = v'$$

- partial *put*: equality test to guarantee injectivity
- for every pair  $(v_1, v)$ ,  $v_1$  can be reconstructed from  $f v$
- similar for *remsnd*

Apply two putlenses to both sides of a pair

$$\forall f : S_1 \Leftarrow V_1, g : S_2 \Leftarrow V_2. f \otimes g : (S_1, S_2) \Leftarrow (V_1, V_2)$$

$$put (s_1, s_2) (v_1', v_2') = (s_1', s_2')$$

$$\text{where } s_1' = put_f s_1 v_1'$$

$$s_2' = put_g s_2 v_2'$$

## Inject a tag in the view (user-specified predicate)

$$\forall p : \text{Either } V \ V \rightarrow V \rightarrow \text{Bool}. \text{inj } p : \text{Either } V \ V \Leftarrow V$$
$$\text{put } s \ v' \mid \text{either } \text{id id } s \equiv v' = s$$
$$\mid \text{otherwise} = \mathbf{\text{if } p \ s \ v' \ \text{then } \text{Left } v' \ \text{else } \text{Right } v'}$$

## Inject a tag in the view (retrieved from the source)

$$\text{injS} : \text{Either } V \ V \Leftarrow V$$
$$\text{injS} = \text{inj } (\lambda s \ v' \rightarrow \text{either } \text{True } \text{False } s)$$

## Ignore tags in the view

$$\forall f : S \Leftarrow V_1, g : S \Leftarrow V_2. f \nabla g : S \Leftarrow \text{Either } V_1 \ V_2$$

$$\text{put } s \ (\text{Left } v_1) = \text{disjoint } f \ g \ (\text{put}_f \ s \ v_1)$$

$$\text{put } s \ (\text{Right } v_2) = \text{disjoint } g \ f \ (\text{put}_g \ s \ v_2)$$

$$\text{disjoint } x \ y \ s \mid (\text{isJust } (\text{get } x \ s)) \wedge \text{isNothing } (\text{get } y \ s) = s$$

- **constraint:** the domains of  $\text{get}_f$  and  $\text{get}_g$  must be disjoint to guarantee injectivity
- **extension** (“observable”  $\text{get}$  domains)

$$\text{data } S \Leftarrow V = \text{PutLens } \{ \text{put} : S \rightarrow V \rightarrow S$$

$$\quad , \text{get} : S \rightarrow \text{Maybe } V \}$$



## Ignore tags in the view (source-based branching)

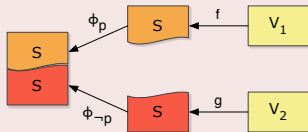
$$\forall p : S \rightarrow Bool, f : S \Leftarrow V_1, g : S \Leftarrow V_2. f \nabla_p g : S \Leftarrow \text{Either } V_1 \ V_2$$

$$f \nabla_p g = (\Phi \ p) \circ f \nabla (\Phi \ (\neg \circ p)) \circ g$$

$$\text{dom } f \ s = \mathbf{case} \ \text{get}_f \ s \ \mathbf{of}$$

$$\{ \text{Nothing} \rightarrow \text{False}; \text{Just } \_ \rightarrow \text{True} \}$$

$$f \bullet \nabla g = f \nabla_{\text{dom } f} g$$

$$f \nabla \bullet g = f \nabla_{\neg \circ \text{dom } g} g$$


## if-then-else view conditionals

$$\forall p : S \rightarrow V \rightarrow Bool, f : S \Leftarrow V, g : S \Leftarrow V. \text{ifthenelse } p \ f \ g : S \Leftarrow V$$

$$\text{ifthenelse } p \ f \ g = (f \nabla_{\phi_{\text{dom } f}} g) \circ \text{inj } p$$

$$\forall p : V \rightarrow Bool, f : S \Leftarrow V, g : S \Leftarrow V. \text{ifVthenelse } p \ f \ g : S \Leftarrow V$$

$$\forall p : S \rightarrow Bool, f : S \Leftarrow V, g : S \Leftarrow V. \text{ifSthenelse } p \ f \ g : S \Leftarrow V$$

## Sums - Disjoint *put* application

Applies two putlenses to distinct sides of a choice

$$\forall f : S_1 \Leftarrow V_1, g : S_2 \Leftarrow V_2. f \oplus g : \text{Either } S_1 \ S_2 \Leftarrow \text{Either } V_1 \ V_2$$
$$\begin{array}{ll} \text{put } (\text{Just } (\text{Left } s_1)) & (\text{Left } v_1') = \text{Left } (\text{put}_f (\text{Just } s_1) v_1') \\ \text{put } s & (\text{Left } v_1') = \text{Left } (\text{put}_f \text{ Nothing } v_1') \\ \text{put } (\text{Just } (\text{Right } s_2)) & (\text{Right } v_2') = \text{Right } (\text{put}_g (\text{Just } s_2) v_2') \\ \text{put } s & (\text{Right } v_2') = \text{Right } (\text{put}_g \text{ Nothing } v_2') \end{array}$$

- **extension** (source value creation)

$$\text{data } S \Leftarrow V = \text{PutLens } \{ \text{put} : \text{Maybe } S \rightarrow V \rightarrow S$$
$$, \text{get} : S \rightarrow \text{Maybe } V \}$$

Inject and “uninject” left/right tags

$$\begin{array}{ll} \text{injl} : \text{Either } V \ S_2 \Leftarrow V & \text{injrl} : \text{Either } V \ S_2 \Leftarrow V \\ \text{uninjl} : V \Leftarrow \text{Either } V \ S_2 & \text{uninjr} : V \Leftarrow \text{Either } V \ S_2 \end{array}$$

## Algebraic data types

$$\text{in}_{[A]} : [A] \Leftarrow \text{Either } () (A, [A])$$

$$\text{nil} : [A] \Leftarrow (), \text{cons} : [A] \Leftarrow (A, [A])$$

$$\text{nil} = \text{in}_{[A]} \circ \text{injl}$$

$$\text{cons} = \text{in}_{[A]} \circ \text{injrl}$$

$$\text{out}_{[A]} : \text{Either } () (A, [A]) \Leftarrow [A]$$

$$\text{unnil} : () \Leftarrow [A], \text{uncons} : (A, [A]) \Leftarrow [A]$$

$$\text{unnil} = \text{uninjl} \circ \text{out}_{[A]}$$

$$\text{uncons} = \text{uninjrl} \circ \text{out}_{[A]}$$

## Products

$$\text{swap} : (B, A) \Leftarrow (A, B)$$

$$\text{assocl} : ((A, B), C) \Leftarrow (A, (B, C)) \quad \text{assocr} : (A, (B, C)) \Leftarrow ((A, B), C)$$

## Sums

$$\text{coswap} : \text{Either } B A \Leftarrow \text{Either } A B$$

$$\text{coassocl} : \text{Either } (\text{Either } A B) C \Leftarrow \text{Either } A (\text{Either } B C)$$

$$\text{coassocr} : \text{Either } A (\text{Either } B C) \Leftarrow \text{Either } (\text{Either } A B) C$$

## Distributivity

$$\text{distl} : \text{Either } (A, C) (B, C) \Leftarrow (\text{Either } A B, C)$$

$$\text{distr} : \text{Either } (A, B) (A, C) \Leftarrow (A, \text{Either } B C)$$

# A point-free put-based bidirectional language (Summary)

## Language of point-free putlens combinators

$Put ::= id \mid Put \circ_{<} Put \mid \Phi p \mid bot p \mid Prod \mid Sum \mid Cond \mid Iso \mid Rec$

$Prod ::= addfst f \mid addsnd f \quad -- \text{create pairs}$   
           $\mid remfst f \mid remsnd f \quad -- \text{destroy pairs}$   
           $\mid Put \otimes Put \quad -- \text{product}$

$Sum ::= inj p \quad -- \text{create choices}$   
           $\mid Put \nabla Put \quad -- \text{destroy choices}$   
           $\mid Put + Put \quad -- \text{sum}$

$Cond ::= ifthenelse \mid ifVthenelse \mid ifSthenelse \quad -- \text{conditional put app.}$

$Iso ::= swap \mid assocl \mid associ \quad -- \text{rearrange pairs}$   
           $\mid coswap \mid coassocl \mid coassoc \quad -- \text{rearrange choices}$   
           $\mid distl \mid distr \quad -- \text{distr. choices over pairs}$

$Rec ::= in \mid out \quad -- \text{algebraic data types}$

## Example (list embedding)

- *put* function

$embedAt :: Int \rightarrow [a] \rightarrow a \rightarrow [a]$

$embedAt\ 0\ (x : xs)\ y = y : xs$

$embedAt\ i\ (x : xs)\ y = x :$

$embedAt\ (pred\ i)\ xs\ y$

- *get* function

$get :: Int \rightarrow [A] \rightarrow A$

$get\ 0\ (x : xs) = x$

$get\ i\ (x : xs) =$

$get\ (pred\ i)\ xs$

$embedAt :: Int \rightarrow [A] \Leftarrow A$

$embedAt\ i = remsnd\ (const\ i) \circ\<\ embedAt'$

$embedAt' :: (Int, [A]) \Leftarrow A$

$embedAt' = ifSthenelse\ (\lambda(i, l) \rightarrow i \equiv 0)\ zero\ nonzero$

**where**  $zero = addfst\ (\lambda s\ v \rightarrow 0) \circ\<\ unhead$

$nonzero = ((+1) \otimes\ untail) \circ\<\ embedAt$

$unhead = cons \circ\<\ keepsnd$

$untail = cons \circ\<\ keepfst$

`get (embedAt 2) "abcd" = Just 'c'`

`put (embedAt 2) (Just "abcd") 'x' = "abxd"`

`put (embedAt 2) (Just "a") 'x' = **undefined`

# Example (DB projection)

- *get* function

**type** *Person* = (*Name*, *City*)

*name* : *Person* → *Name*

*city* : *Person* → *City*

*peopleNames* : [*Person*] → [*Name*]

*peopleNames* = *map name*

- *put*-based lens

*map* :  $B \Leftarrow A \rightarrow [B] \Leftarrow [A]$

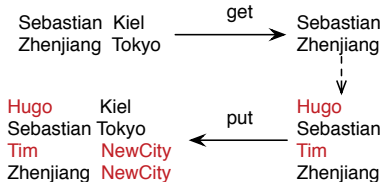
*map f* = *ifV* *then* *else null* (*nil* ∘ *unnil*) *it*

**where** *it* = *cons* ∘  $(f \otimes \text{map } f)$  ∘ *uncons*

*peopleNames*<sub>0</sub> : [*Person*] ⇐ [*Name*]

*peopleNames*<sub>0</sub> = *map* (*addsnd cityOf*)

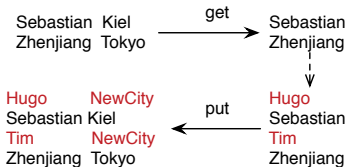
**where** *cityOf s v* = *maybe* "NewCity" *id s*



# Example (DB projection with environment)

- *put*-based lens

```
peopleNames : [Person] <-_E [Name]
peopleNames = withMbS peopleNames'
peopleNames' : [Person] <-_{[Person]} [Name]
peopleNames' = map (addsnd cityOf)
  where cityOf people n =
    case lookup n people of
      Just c → c
      Nothing → "NewCity"
```



- **extension** (global environment)

```
data S <-_E V = PutLens { put : Maybe S → V → Reader E S
                        , get : S → Maybe V }
```

```
addsnd : (E → A → B) → (A, B) <-_E A
withMbS : (S <-_{Maybe S} V) → (S <-_E V)
withMbV : (S <-_{Maybe V} V) → (S <-_E V)
withV'   : (S <-_V V) → (S <-_E V)
```

# Example (DB projection with state)

- *put*-based lens

$\text{peopleNames}_1 = \text{initSt} (\lambda e v \rightarrow 0)$

$\text{peopleNames}_1' : [Person] \leftarrow_{[Person]}^{Int} [Name]$

$\text{peopleNames}_1' = \text{map}$

$(\text{updateSt } \text{upd} (\text{addsnd } \text{cityOf}))$

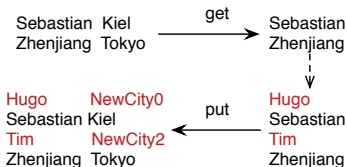
**where**  $\text{cityOf } i \text{ people } n =$

**case**  $\text{lookup } n \text{ people of}$

$\text{Just } c \rightarrow c$

$\text{Nothing} \rightarrow \text{"NewCity"} \text{ ++ } \text{show } i$

$\text{upd } i \text{ e s} = i + 1$



- **extension** (state)

**data**  $S \leftarrow_E^{St} V = \text{PutLens} \{ \text{put} : \text{Maybe } S \rightarrow V \rightarrow \text{ReaderT } E \text{ (State } St) S$   
 $, \text{get} : S \rightarrow \text{Maybe } V \}$

$\text{initSt} : (E \rightarrow V \rightarrow St) \rightarrow (S \leftarrow_E^{St} V) \rightarrow (S \leftarrow_E V)$

$\text{updateSt} : (St \rightarrow E \rightarrow S \rightarrow St) \rightarrow (S \leftarrow_E^{St} V) \rightarrow (S \leftarrow_E^{St} V)$



- a novel point-free put-based BX language
  - we propose to shift into a *put* programming style
    - users write well-behaved *put*
    - language provides unique *get* for free
  - *put programming is not easier, but rather more powerful*
  - this shift is **manageable**
    - the combinators encapsulate different *put* behaviors
    - complex *put* behaviors by composition (and using extensions)
  - this shift is **necessary**
    - full control of the backward transformation (user's intentions)
- + more expressive than existing total get-based languages
- + better updatability than existing partial get-based languages

## Demos: Haskell++

- <http://hackage.haskell.org>  $\Rightarrow$  putlenses
- synthesize more efficient *put* and *get* functions
- point-free VS point-wise: translate higher-level functional put programming language to lower-level core language
- languages for other domains (e.g., lenses for relational data)



A. Bohannon, B. C. Pierce, and J. A. Vaughan

Relational lenses: a language for updatable views

*Principles of Database Systems, 2006.*